

Leif B. G. Andersen and Vladimir V. Piterbarg: Interest Rate Modeling

Atlantic Financial Press, approx. 298 USD, 3 volumes:

- **Volume 1: Foundations and Vanilla Models, 492 pages**
- **Volume 2: Term Structure Models, 376 pages**
- **Volume 3: Products and Risk Management, 546 pages**

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One would expect that more than 1400 pages on interest rate modeling would provide a comprehensive and overwhelming treatment of the subject. These three volumes definitely meet that expectation, spanning everything from very general, introductory topics such as principal component analysis in the first volume to very specific, computational time-efficient valuation questions in Volume 3.

The authors have managed an ideal combination of research and application, no doubt at least partially due to their distinguished credentials in both areas: both have published in leading academic journals on interest rate models; both occupy managing director positions in Wall Street investment banks. Although style, notation, and level of abstraction are equal across the three volumes, each book has its specific focus.

Volume 1: Foundations and Vanilla Models, 492 pages

The authors do not restrict themselves to the subject of interest rates in this first volume, but cover a wide range of topics, for example, equivalent martingale measures and arbitrage pricing, Girsanov's Theorem, and Black–Scholes. Even this first book is definitely not on the introductory level. Readers who have worked through Hull should be just about ready for the level of complexity encountered in Volume 1 of *Interest Rate Modeling*. Two of the longer chapters are devoted to finite difference and Monte Carlo methods, clearly indicating the direction the trilogy takes toward the numerical methods necessary for the pricing and risk management of

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fixed income derivatives. Interest rate models are mentioned for the first time in Chapter 4. The useful probability measures are presented and an introduction to the Heath/Jarrow/Morton framework, along with its most important representations, is made. The instruments chapter is next, having perhaps a too short introduction to the markets and their participants. Unfortunately, in this section, the literature citations are more than 10 years old. Including references to several homepages would have provided the reader with more up to date information. The section on instruments, however, is very comprehensive and covers not only the standard derivatives but also capped and floored floaters, digitals, multi-rate products, and range accrual notes. A discussion of targeted redemption notes (TARNs) and volatility derivatives is also included, preparing the ground for their valuation in Volume 3.

The second part *Vanilla Models* starts with a chapter on the yield curve and state-of-the-art approximation methods. Chapter 7 covers single-factor models in which volatility is a deterministic function of the interest rate. The next logical step is an extension to stochastic volatility, time-homogeneous, and time-dependent models. The authors again emphasize implementation, introducing the Fourier transformation and addressing calibration issues.

Foundations and Vanilla Models does not really work as a stand-alone book because it is full of pieces that will only come together in a unified picture in the next two volumes of the trilogy. However, it could serve as a very useful reference in understanding what is to come.

Volume 2: Term Structure Models, 376 pages

The second volume really embeds itself in the interest rate environment. It covers the affine term structure models, especially the Gaussian case using the older Ho/Lee and Vasicek models. The extension to models preventing negative interest rates and introducing interest-rate-dependent short rates is easy to understand. Chapter 11 covers the log-normal short rate models, especially Black/Derman/Toy and Black/Karasinski. The comparably new concept of unspanned stochastic volatility is briefly explained with the (negative) example of the Fong/Vasicek model. Why the authors chose the models they did for this part of the book is not completely clear since, e.g., the Hull/White model is not covered. An overview of models, perhaps presented in a table, would have helped the reader classify and clarify the virtual “zoo” of models discussed. Very helpful are the links back to the general Heath/Jarrow/Morton framework in Volume 1. Multi-factor short rate models in Chapter 12 are probably covered for the sake of completeness, but between the one-factor models that are so simple and quick to use and the LIBOR market model workhorse, the next generation of interest rate modeling textbooks might omit a discussion of multi-factor short rate models without risk of criticism. The same holds for the quasi-Gaussian models in the following chapter. The remainder of Volume 2 is devoted to the LIBOR market model and is easy to comprehend. Starting with the standard setup, much space is devoted to modeling the correlation structure. In this section, the authors provide a good example based on data from U.S. interest rates. The pricing of some standard derivatives leads the reader to the calibration and Monte Carlo simulation sections

with suggestions for efficient implementation. The last chapter of Volume 2 covers some practical issues in LMM implementation, such as, for example, interpolation, and introduces the swap market model as well as other extensions of the LMM.

This book, in contrast to so many others, devotes a great deal of space to calibration and implementation questions, which draws attention to real-life problems and makes theory quickly applicable. This is true not only for models having closed-form solutions (mainly affine term structure models) but also for the more sophisticated models requiring numerical methods.

A large part of Volume 2 is about specific cases of the general Heath/Jarrow/Morton model covered in Volume 1. This structure makes Volume 2 (like Volume 1) a book that cannot be read on its own. Furthermore, it would be difficult to use this term structure modeling book in class since the important unifying approach is covered in *Foundations and Vanilla Models*.

Volume 3: Products and Risk Management, 546 pages

The final volume takes a different perspective: no longer does the model hold center stage; instead, products (in Part IV) and problems of risk management (Part V) play the leading roles, while models are selected as useful tools for solving challenges.

The single-rate vanilla derivatives chapter considers very simple derivatives or building blocks, such as caps, floors, and European swaptions. In their treatment of such a specific topic as the modeling of the terminal swap rate, the authors once again show the practical application of the book. Furthermore, the vanilla chapter covers products that are, perhaps, not so very vanilla, such as LIBOR in arrear swaps, LIBOR with delay swaps, and constant maturity swaps (CMS). The section on CMS is particularly valuable since it shows the pluralism of methods available to deal with this specific product. Chapter 17 turns to the European style multi-rate vanilla derivatives, for which the LMM has some real advantages. After a brief overview of the basic copulas, the first application is CMS spread options. Once again, there is an emphasis on implementation and a practical view of the advantages and shortcomings of copulas in interest rate derivatives valuation. The stochastic volatility models reveal an alternative way of accounting for the interdependence of interest rates. In Chapter 18 on callable LIBOR exotics, the focus is clearly on Monte Carlo methods, using a LIBOR market model and a number of algorithms for efficient implementation. Only for a few applications is the local projection method of simpler models for more speed possible. Chapter 19 on Bermudan swaptions considers a specific case of callable products. The main focus is on tractability and, therefore, on local projection methods using low-dimensional models. However, the specific simplifications that are possible in Monte Carlo valuation are covered as well. The remainder of Part IV covers exotic products such as TARNs, volatility swaps, and forward swaption straddles, as well as adjustments to match model prices with market prices. Some calibrations of implied volatilities might not lead to market-consistent prices of the underlying, which makes an adjustment of either model, market, or trade inevitable.

The part on risk management is essentially on the calculation of sensitivities, i.e., the Greeks. Starting from the simplest Black/Scholes Greeks, the authors go

on to smoothing techniques that are inevitably necessary for complex derivatives. The chapter on path-wise differentiation is a specific application to callable or barrier products. Importance sampling and control variates are another approach to obtain reliable Greeks. Very useful is the connection of methods to instruments and the discussion of advantages and disadvantages. The last chapter of the three volumes is devoted to the estimation of vega, specifically in LIBOR market models. The authors show the connection of calibration technique and vega calculation with a nice example, once again reflecting the applied focus of the book.

Volume 3 of *Interest Rate Modeling*, in contrast to the other two volumes, can be read as a stand-alone book. For a reader with a background in advanced modeling, *Products and Risk Management* will provide many useful suggestions for application of models and algorithms to specific products and hedging.

All three volumes contain very valuable lists of literature. Although the contents of *Interest Rate Modeling* might be slightly biased toward the authors' own research, it would be an excellent addition to the reference library of anybody working in the fixed income quantitative environment, whether out in the "real world" or in academia.

Why the authors structure the three volumes the way they did is a bit of a mystery. For example, the Heath/Jarrow/Morton model is covered in Part I: Foundations. Why is it not in Part II: Vanilla Models or, alternatively, why isn't it in Part III: Term Structure Models as is the Libor market model?

Compared to another leading textbook in the field, Brigo/Mercurio's *Interest Rate Models—Theory and Practice*, the book reviewed here is more comprehensive in the sense that it covers a wider range of both basic and advanced topics (and is, consequently, about 400 pages longer). The book has a clear focus on interest rates, spending less time, if any, on other topics, for example, credit risk.

Interest Rate Modeling may not be the best choice of textbook for a course on advanced modeling. The topics are not ordered from easy to difficult and studying all three volumes could well take more time than one course provides. However, *Interest Rate Modeling* is a very valuable aid to understanding the implementation problems of theoretically beautiful models and it covers many specific problems as well as their solutions. We look forward to a second edition with code that makes the algorithms even more hands-on.